Advanced Level Further Mathematics

## JUNE 2004

a) A particle of mass 2 kg moves so that its position vector at time

1 seconds is given by

$$
r=[t-\sin t) i+(1-\cos t) j] m \text {. Find }
$$

the magnitude of its acceleration when $t=\pi / \mathbf{2}$
the magnitude of the force acting on the particle when $t=\pi / 2$

## for $0<t<\pi / 2$ the time when its acceleration is parallel to the vector $i$ +j .

(ii) A force of magnitude $\left(4+v^{2}\right) / 10 \mathrm{~N}$ acts on a particle of mass 5 kg so that it moves in a straight line from rest . find to one decimal place, its speed after $5 \pi$
seconds.
b) A projectile is fired from a point on the ground 10 m away from a vertical wall and it just clears the wall and another parallel wall. The distance between the walls is 20 m , and the height of each wall is 2 m . The plane in which the projectile travels is perpendicular to the planes of the walls. Find the angle above the horizontal at which the projectile was fired, and the greatest height above the Walls attained by the projectile.
Find also the total time of flight of the projectile and the range on the horizontal plane through the point of projection.
[Take g as $10 \mathrm{~ms}-2$ ]
4. Two forces, $F_{1}$ and $F_{2}$ where $F_{1}=(5 i+2 j+7 k) N, F_{2}=(4 i+j-k) N$, act at points whose position vectors are $(-3 \mathrm{i}+3 \mathrm{j}-2 \mathrm{k}) \mathrm{m}$ and $(10 \mathrm{i}+7 \mathrm{j}+3 \mathrm{k}) \mathrm{m}$ respectively. Show that the line of action of these force intersect. A third force $F_{3}$ acts so that the system of forces $F_{1}, F_{2}$ and $F_{3}$ is in equilibrium. Find the magnitude of $\mathrm{F}_{3}$, and an equation of its line of action.
ii) A train is uniformly retarded from a speed of $25 / 3 \mathrm{~m} \mathrm{~s}^{-1}$ to a speed of $25 / 9 \mathrm{~m} \mathrm{~s}^{\prime}{ }^{1}$. It travels some
distance at the latter speed and is then uniformly accelerated until its speed is again $\mathrm{m} \mathrm{s}^{-1}$, the magnitude of the acceleration being half the magnitude of the retardation. The time from the beginning of the retardation until the speed is again $25 / 3 \mathrm{~m} \mathrm{~s}^{1}$ is 450 seconds and the total distance covered is 2000 m . Find the distance travelled at $25 / 3 \mathrm{~m} \mathrm{~s}^{\prime 1}$.
5. Three smooth spheres $A, B, C$ of mass $m, 2 m, 4 m$ respectively, rest on a smooth
horizontal table with their centres collinear and B lying between A and C . The coefficient of restitution between $A$ and $B$ and between $B$ and $C$ is $e . A$ is projected directly towards $B$ with velocity
u , and C moves with velocity $\mathrm{u} / 4$ after it has been struck by B. Find the value of $e$ and show that $A$ and $B$ are reduced to rest .
(a) (i) A uniform beam AB of length $A$ and weight W is free to turn in a vertical plane about a hinge at $A$ and is supported in horizontal position by a string attached to the beam at a point D at a distance $\mathrm{a} / 3$ from A and to a point F at height $b$ vertically above A show that the tension in the string is
$\left(\mathbf{W} \sqrt{ } \mathbf{a}^{2}+9 \mathrm{~b}^{2}\right) / 2 \mathrm{~b}$
Find in term of $W, A$ and $B$ the magnitude of the reaction at the hinge. Also find the tangent this reaction makes with the horizontal.
7. A particle of mass $m$ lies on a smooth horizontal table, and is attached by an inextensible string which passes through a smooth hole in th - table, to a particle of mass 2 m which hangs freely below the table. The particle of mass m describes a circle of radius 1 m on the table with such uniform speed that the parti rie of mass 2 m remains at rest. Calculate the uniform speed.[Take $g$ as $10 \mathrm{~ms}^{\prime 2}$ ]
(a) (i) An engine working at a constant rate of 50 kW pulls a train of mass $20,000 \mathrm{~kg}$ along a level track against a constant non-gravitational resistance of 1600 N . Find the acceleration of when the speed is $25 / 2 \mathrm{~ms}^{-1}$
The train now moves up a line of grestest slop of a plane of inclination $\sin ^{-1}(1 / 100)$ to the horizontal. find the maximum speed of the train up this plane
(ii) Two light springs AB and BC each of natural length 0.1 m and of modulus of elasticity SON and 40 N respectively are joined at B to form one spring $A B C$. The spring is suspended at $A$ and a mass of 2 kg hangs freely at C so that the combined spring is vertical. Find the new length of the combined spring.
(b) (i) $10 \%$ of a lams consignment of oranges is known to be bad. If three oranges are chosen at random from the consignment, find the probability that
8. all will be bad,
9. none will be bad
10. at least one will be bad.
(ii) The probability ${ }^{7}$ that a man makes a journey by $\operatorname{Car}(\mathrm{C})$, motorbike (M) or on foot (F)
are $1 / 2,1 / 6,1 / 3$ respectively. The probability of having accident (A) when he choose these means of transport are $1 / 5,3 / 5,1 / 10$ respectively. Find the the probability
(a) of having an accident,
(b) that he made the journey by motor-bike, given that he had an accident,
(c) he had an accident or he made the journey by car.

