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Advanced Level Further Mathematics  
Paper Three

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a) At time  $t$  seconds, the velocities  $v_1, v_2$  of two particles  $P_1, P_2$  respectively, are given by

$$V_1 = (2ti - 3t^{1/2}j) \text{ ms}^{-1}$$

$$v_2 = [t^3i + (21 - 3t)j] \text{ ms}^{-1}$$

Find the non-zero value of  $t$  for which the accelerations of  $P_1$  and  $P_2$  are perpendicular.

Obtain the velocity of  $P_1$  relative to  $P_2$  when their accelerations are perpendicular.

Given that  $P_1$  and  $P_2$  are at the origin when  $t = 0$ , find

The distance between  $P_1$  and  $P_2$  when their accelerations are perpendicular.

2.

b) Find the abscissa of the centroid of the finite region bounded by the curve  $y = \ln x$  and the lines  $y = 0$  and  $x = 2$ , giving your answer correct to 2 decimal places..

c) A uniform rod  $AB$  of weight  $W$ , rests in equilibrium with the end  $A$  against a smooth vertical wall

and the end  $B$  on rough horizontal floor. Given that  $AB$  makes of  $45^\circ$  with the floor, find, in terms of  $W$ , the forces exerted on the rod at  $A$  and  $B$ . Hence show that if  $\mu$  is the coefficient of friction between the rod and the floor, then  $\mu \geq \frac{1}{2}$

3.

4. A particle starts from rest at point  $O$  and moves in a straight line with a constant acceleration of  $2 \text{ ms}^{-2}$ . Three seconds later, a second particle starts from rest at the same point  $O$ , and moves after the first particle with constant acceleration  $4 \text{ ms}^{-2}$ . Find how much time elapses before the second particle overtakes the first.

5. The magnitude of the retardation to the motion of a particle moving along the  $x$  — axis is  $\frac{1}{2}v^2 \text{ ms}^{-2}$ , where  $v \text{ ms}^{-1}$ , is the speed of the particle. At time  $t=0$ , the particle passes through the origin at a speed  $u \text{ ms}^{-1}$  in the direction of increasing  $x$ . Show that the speed of the particle is halved when  $t = 7u^2/12$  and find, in terms of  $u$ , the value of  $x$  at this instant.

4. Referred to horizontal and vertical axes  $OX$  and  $OY$  respectively, a particle is projected from the point  $O$  with a velocity of  $65 \text{ ms}^{-1}$  at an angle elevation of  $\cos^{-1}(12/13)$

ii) Show that the particle passes through the point  $A(60, 20)$  and  $B(120, 30)$ .

iii) Find the velocity of the particle  $B$  and the further horizontal distance covered before the particle is again at the level  $B$ .

(Take  $g$  as  $10 \text{ ms}^{-2}$ )

(a) Two particles  $P$  and  $Q$ , of mass  $2m$  and  $5m$  respectively, are connected by a light inextensible string. Particle  $P$  lies on a horizontal table and the string passes over a small smooth fixed pulley at the edge of the table, so that particle  $Q$  hangs freely at the other end of the string, at a distance  $2l$  above the ground.

The coefficient of friction between  $P$  and the table is  $\frac{1}{2}$  and the frictional force between the string and the table is negligible. The system is released from rest, with both parts of the string taut and the hanging part vertical. Find

7. The tension in the string and the acceleration of the system,

(a) The magnitude and direction of the force exerted on the pulley. Show that

c. The speed of  $Q$  just when it has descended a distance  $l$  is  $\sqrt{\frac{8gl}{2}}$  and that the time taken is  $\sqrt{\frac{2l}{2g}}$

Just after  $Q$  has descended through a distance  $l$  the string is cut.

Find

8. The speed at which  $Q$  eventually strikes the ground, giving your answer in terms of  $g$  and  $l$ .  
[assuming that  $P$  does not reach the edge of the table.]

(a) Three spheres  $S_1, S_2$  and  $S_3$  of each of mass  $m$ , lie in a straight line with  $S_3$  between  $S_1$  and  $S_2$ ,  $S_1$  impinges directly on  $S_2$  with speed  $u$  and  $S_2$  then impinges directly on  $S_3$ . The coefficient of restitution is  $e$  for all impacts involved. Calculate in terms of  $u$  and  $e$ , the speeds of

(ii)  $S_1$  and  $S_2$  after their impact

(iii)  $S_2$  and  $S_3$  after their impact

Show that the loss in kinetic energy during the impact between  $S_1$  and  $S_2$  is  $\frac{1}{4} mu^2(1-e)(1+e)$

(f) The constant non-gravitational resistance to the motion of a car of mass  $1500 \text{ kg}$  is  $R \text{ N}$ . The engine of the car works at a constant rate of  $20 \text{ kW}$ . Given that the maximum speed of the car up a plane of inclination  $\sin^{-1}(1/10)$  to the horizontal is  $10 \text{ ms}^{-1}$ , find

i) The value of  $R$ ?

ii) The maximum speed of the car on level ground.

Find also the speed of the car when it is accelerating at the rate of  $2 \text{ ms}^{-2}$

i) Down the plane

ii) Up the plane

iii) On the plane (Take  $g$  as  $10 \text{ ms}^{-2}$ )

8.

a Three balls are, to be drawn at random and without replacement from a bag containing 16 balls of which 4 are red. Find the probability that

a) All three balls will be red,

ii) None of the three balls will be red,

b) At least one of the balls will be red,

c) Exactly one of the balls will be red.

ii. A die with faces numbered 1 to 6 is biased so that the probability of any even face is twice the probability of any odd face. The die is to be thrown twice. Find

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- d) The probability that the total score will exceed 10.

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