# A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 1999 MEETLEARN.COM



*A Level Further Pure Maths*

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1. The curve C is given by the parametric equations $x = a(t - \sin t), y = a(1 - \cos t), a > 0, 0 \le t \le 2\pi$.
Sketch the curve C. show that the area of the finite region bounded by the curve C and the x – axis is $3\pi a^2$. Deduce the mean value of y with respect to x for $0 \le t \le 2\pi$. The curve C is rotated completely about the x – axis.

Show that the area of the surface generated is $\frac{64\pi a^2}{3}$

2. A linear transformation L is represented by matrix A and another transformation is represented by the matrix B.

Given that $A = \begin{pmatrix} 5 & 7 & 2 \\ 2 & 3 & 1 \\ -1 & -2 & 0 \end{pmatrix}$ *and* $B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -5 & -2 & 1 \end{pmatrix}$, find

a. $A^{-1}$

b. A Cartesian equation of the image of the line $\frac{x-1}{2} = \frac{y}{-2} = \frac{z}{1}$ under the transformation L.

c. the vector x such that $(AB)x = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

3. (i) Determine the value of $\lambda$ for which $y = \lambda x e^{-x}$ is a particular integral of the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^{-x}$. Find the general solution of this differential equation.

(ii) Solve the differential equation

$$\cos x \frac{dy}{dx} + 2y \sin x = \sin^2 x \cos x, 0 < x < \frac{\pi}{2}, \text{ given that } y = -\frac{\pi}{8} \text{ when } x = \frac{\pi}{4}$$

4. (a) Given that $z = \cos\theta + i\sin\theta$, show that $z^n + z^{-n} = 2\cos n\theta$ *and* $z^n - z^{-n} 2i\sin n\theta$.
Hence or otherwise,

show that $\cos^4 2\theta + \sin^4 2\theta = \frac{1}{4}(\cos 8\theta + 3)$

(b) The transformation $T : z \to \omega$ in the complex plane is defined by $\omega = \frac{az+b}{iz+c}$.

Given that $\omega = 2 - 2i$ *when* $z = 1 - i$ *and* $\omega = 2 - 6i$ *when* $z = 1 - i$, find the values of the real constants a, b, and c

5. Prove that the equation of the tangent at the point $P\left(ct, \frac{c}{t}\right)$ on the rectangular hyperbola $xy = c^2$ *is* $x + t^2 y = 2ct$. This tangent meets the x – axis at Q and the y – axis at R. prove that P is the midpoint of QR. The line through Q parallel to y – axis meets the hyperbola at S, and the line through R parallel to the x – axis meets the hyperbola at T. prove that the areas of triangles PQS and PRT are equal. Prove, also that as P varies, the locus of the midpoint of ST is the rectangular hyperbola $16xy = 25c^2$

6. Using the definitions of the hyperbolic sine and the hyperbolic cosine, prove that

$$\tanh^{-1} x = \frac{1}{2} \ln\left[\frac{1+x}{1-x}\right], -1 < x < 1.$$

a. Sketch the curve $y = \tanh^{-1} x$.