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A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2000 MEETLEARN.COM

A Level Further Pure Maths

1. (i) Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$. Find the real values of x for which $\cosh^2 x + \sinh^2 x = 3$, giving your answer correct to three places of decimals

(ii) Given that n is a non-negative integer and that $I_n = \int_0^1 x(1 - x^3)^n dx$,

show that for n>0, $(3n + 2)I_n = 3nI_{n-1}$, hence evaluate I_5

2. (a) Solve for real values of x the inequality $|2x - 4| - |x + 2| > 2$

(b) The vectors X_1 and X_2 are given by $X_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, X_2 = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$.

The vector $X_3 = \begin{pmatrix} \lambda \\ 1 \\ \mu \end{pmatrix}$, $\lambda > 0$ is perpendicular to the vector X_1. The modulus of X_3 is $\sqrt{11}$.

Find λ and μ

Given that $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 3 & 1 & 4 \end{pmatrix}$.

Find A^{-1}. Hence or otherwise, solve for Y_1, Y_2, and Y3 given that $X_i = AY_i$, i = 1,2,3

3. (a) Prove that the line $x - ty + at^2 = 0, (a \neq 0)$, is a tangent to the parabola $y^2 = 4ax$ for all real values of t and find the coordinates of the point of contact.

(b) Show that the equation of the chord joining the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ is $(p + q)y - 2x = 2apq$.
Given that this chord passes through the focus, show that $pq = -1$ and that the focus of M, the midpoint of PQ, is another parabola and find the coordinates of its locus

4. Obtain the equation of the tangent at the point with parameter t, to the curve C whose parametric equations are given by $x = a\cos^2 t, y = a\sin^2 t, 0 \le t \le \frac{\pi}{2}, a > 0$

Show that if this tangent meets the coordinates axes at A and B, then AB is of constant length.
Sketch the curve C. find the area of the finite region enclosed by the curve and the coordinate axes.

[You may use the fact that $\int_0^{\frac{\pi}{2}} \sin^n x dx = I_n = \left(\frac{n-1}{n}\right) I_{n-1}$]

5. Show that lines L_1 and L_2 with vector equations $L_1 : r = 13i + 4j + 11k + \lambda(4i - 8j - 6k)$, $L_2 : r = 5i + 22j + 9k + \mu(7i - 17j - 5k)$, where λ and μ are real constants, intersect and find the coordinates of the point of intersection. Find also, in the form $r.n = p$, the vector equation of the planes Π containing L_1 and L_2. Find a Cartesian equation of the plane Π_1, which is parallel to Π and contains the point Q with coordinates (- 2, 4, 1)

6. (i) The function f is given by
$f(x) = \frac{x^3 + 5x^2 - 6x - 30}{x + 5}, x \neq -5$, define f at x = -5 so that f is continuous at this point
(ii) Given that $y'' + y' - 2y = x^2$, find constants a, b, c so that $y = ax^2 + bx + c$ is a particular integral to this differential equation. Find the solution of the differential equation $y'' + y' - 2y = x^2$ when satisfies the conditions $y' = 0$ and y = 1 when x = 0

7. (a) Express $f(x) = \frac{x+1}{(2x+1)^2(4x^2+1)}$ in partial fractions.

Hence, or otherwise show that $\int_0^1 f(x)dx = \frac{1}{4}\ln\left(\frac{3\sqrt{5}}{5}\right) + \frac{1}{8}\arctan 2 + \frac{1}{12}$

(b) Use De Moivre's theorem to
(i) Show that the polynomial $f(z) = (\cos\alpha + z\sin\alpha)^n - \cos n\alpha - z\sin n\alpha$, where α is real and n is a non zero natural number, is divisible by $z^2 + 1$
(ii) Show that if $\alpha \in \Re$, then $\left(\frac{1 + i\tan\alpha}{1 - i\tan\alpha}\right)^n = \frac{1 + i\tan(n\alpha)}{1 - i\tan(n\alpha)}$

8. Consider the infinite set S of matrices of the form $\begin{pmatrix} b & 0 \\ 0 & 1 \end{pmatrix}, b \in Q$, where Q is the set of rational numbers, under the operation of matrix multiplication. Show that the operation is both commutative and associative. Verify the remaining group properties and hence, show that S does not form a group under matrix multiplication. State the restriction on the rational numbers Q so that S should form an Abelian group. Given that $b \in \Re, b - 0$, does S form a group under matrix multiplication? Give a reason for your answer.

9. (i) Find the expansion of $(\cosh x)ln(1 - 2x)$ in ascending powers of x as far as and including the term in x^3. For what values of x is the expansion valid?
(ii) The curve C_1 and C_2 have polar equations given by
$C_1 : r^2 = a^2\sin 2\theta, 0 \leq \theta \leq 2\pi$ $C_1 : r = a\cos\theta, \frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}, a > 0$
(a) Sketch on the same diagram the curves C_1 and C_2, showing the tangents at the pole.
(b) Find the polar coordinates of the points of intersection of C_1 and C_2.