A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2001 MEETLEARN.COM

A Level Further Pure Maths

1. (i) Given that $1+\sqrt{3}i$ is a root of the equation $z^3+z^2+az+b=0$, find the values of the real numbers a and b. hence, solve the equation.
(ii) Given that Z_1 and Z_2 are two complex numbers, show geometrically or otherwise that
(a) $|z_1+z_2|\geq|z_1-z_2|$, and
(b) $|z_1-z_2|\geq|z_1|-|z_2|$.
(c) Hence, find the least and greatest values of $|z_1+z_2|$, when $z_1=-3+4i$ and $|z_3|=10$

2. (i) Express f(x) where $f(x)=\dfrac{x^2}{(x-1)^2(x^2+1)}$ in partial fractions. Hence, find $\int f(x)dx$.
(ii) Define $\operatorname{cosech} y$ in terms of e^y.
Given that x > 0, prove that $ar\cos echx = \ln\left(\dfrac{1+\sqrt{x^2+1}}{x}\right)$.
Solve the equation $ar\cos ech(2x)+\ln x=\ln 5$.

3. Given the matrix M, where $M=\begin{pmatrix}2 & k & 2\\ 6 & k & 2\\ 4 & -3 & k\end{pmatrix}$
(i) Find the real values of k for which M is invertible.
(ii) For k = 0, find
(a) the image of the line $2i+3j+5k+\lambda(i+j+k$ under the transformation with matrix M.
(b) M^{-1}.
(c) the point whose image is (- 12, 0, 24), under the transformation with matrix M.

4. (i) Sketch, using the same axes, the curves $y^2=4x$ and $y^2=x^3$.
Shade the region for which $(y^2-4x)(y^2-x^3)\leq 0$.
(ii) Show that the polar equation of the curve C with Cartesian equation
$(x^2+y^2)^2=2a^2xy, a>0$ is $r^2=a^2\sin 2\theta$.
Sketch the curve C, showing clearly the tangents at the pole. Verify that the tangent to C at the point where $\theta=\dfrac{\pi}{3}$, is parallel to the initial line.

5. (i) Show that the arc length of the curve $ay^2=x^3$, for $0\leq x\leq\dfrac{7a}{3}$ is $\dfrac{13a}{13}$
(ii) Given that $x=a\sin 2t$, $y=a\cos 2t$, find the mean value of y in the interval $0\leq t\leq\dfrac{\pi}{4}$
(a) With respect to t,
(b) With respect to x.

6. Find the equation of the tangent and the normal at the point $P\left(2a+2t,\dfrac{at^2}{2}\right)$ to the parabola $(x-2a)^2=2ay$.
The tangent and normal at P cut the x - axis at T and N respectively. Prove that $\dfrac{PT^2}{TN}=at$.
Find the coordinates of the point Q at which the normal at P intersect the parabola again.

7. (i) A particular integral of the differential equation
$\dfrac{d^2x}{dt^2}-5\dfrac{dx}{dt}+6x=2\sin 2t$ is $A\sin 2t+B\cos 2t$
Find the values of the real constants A and B. Hence, solve this differential equation giving that when $t=0, x=0$ and $\dfrac{dx}{dt}=1$

(ii) The function f is defined on the set of real numbers by $f(x) = x + [x,]$, where $[x]$ means the greatest integer less than or equal to x. sketch the graph of f for $0 < x < 0$.

Evaluate $\int_0^2 f(x)dx$.

8. (i) the Points A, B, C have position vectors a, b, c, respectively relative to the origin O, where $a = i + 2j + 3k$, b = 2i - 4k, c = 5i - j - 3k. Find

(a) the vector equation of the plane ABC in the form $r.n = p$.

(b) the area of triangle ABC.

(c) the perpendicular distance of the point C from the line AB.

(d) the volume of the tetrahedron OABC.

(e) Evaluate $\int_0^1 \sinh^{-1} 2x dx$, giving your answer to 4 decimal places.

9. (i) Given that every element x of a group (H, *) satisfies the relation x * x = e, where e is the identity element, prove that (H, *) is a commutative group.

(ii) The elements of the set G are ordered pairs (a, b) in Z_2 X Z_3, where Z_2 ={0, 1} and Z_3 ={0, 1, 2}.

Write down the six elements of G.

A binary operation * on G is given by $(a,b)*(c,d) = (a +_2 c +_3 d)$, where $+_2$ and $+_3$ denote addition modulo 2 and addition modulo 3 respectively. Show that $(G,*)$ forms a commutative group.

[you may assume associativity]