002

A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2002 MEETLEARN.COM

A Level Further Pure Maths

1. (i) Evaluate $\int_0^{\ln 2} \cosh^3 x\,dx$

(ii) Given that $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x\,dx$, prove that for the integer $n \geq 2$, $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$

Evaluate I_3 and I_4

2. Find the length of the arc of the curve with parametric equations $x = at^2; y = 2at, a > 0$ between the origin O and the point P with coordinates (a, 2a).
Find, also, the area of the surface of revolution formed when this arc is rotated through 2π radians about the x – axis. Hence, or otherwise, find the y – coordinate of the centroid of the arc OP.

3. (i) A linear transformation $T : \Re^3 \to \Re^3$, has matrix A, where $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 1 & 3 \\ -1 & -2 & 4 \end{pmatrix}$.

Show that every point is mapped onto the plane $5x - 3y + z = o$.
Find the image under T of

(a) the line $\frac{x-2}{2} = \frac{y+1}{-3} = \frac{z-1}{-1}$

(b) the plane $x + y - z = -1$

(ii) Given that G is the set of all cube roots of unity, show that G forms a group under ordinary multiplication of complex numbers.

4. The points S_1 and S_2 have Cartesian coordinates $\left(\frac{-\lambda}{2}\sqrt{2}, 0\right)$ and $\left(\frac{\lambda}{2}\sqrt{2}, 0\right)$, where $\lambda > 0$. Find a Cartesian equation of the ellipse which has S_1 and S_2 as its foci and a major axis of length 4λ.
Write down an equation of a directrix of this ellipse. Given that the parametric equations of the ellipse are $x = \lambda\cos\theta; y = b\sin\theta$, express b in terms of λ.

The point P is given by $\phi = \frac{\pi}{4}$ and the point Q by $\phi = \frac{\pi}{2}$. Find the equation of the chord PQ.

5. (i) Solve the differential equation $(x+1)\frac{dy}{dx} - 3y = (x+1)^5$, given that $y = \frac{3}{2}$ when x = 0

(ii) Determine the constants p and q such that $y = pe^x + qe^{2x}$ is a particular integral of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^x(1+2e^x)$. Hence, or otherwise, find the general solution of this differential equation.

6. (i) Find each of the three cube roots of -1 in the form $a + ib$, where $a, b \in \Re$. Hence, or otherwise solve, for z, the equation $\frac{z^3}{(z-1)^3} = -1$, giving your answers in the form $a + ib$, $a, b \in \Re$.

(ii) Transformation $T : z \rightarrow \omega$ of the complex plane is defined by $\omega = \frac{z-i}{z+i}$.

Points P and Q represent the complex number $z = x + iy$ and $\omega = u + iv$, in the z – plane and the w – plane, respectively. Given that the locus of P is the x – axis, find the Cartesian equation of the locus of Q and sketch the locus of Q on an Argand diagram.

7. The points A, B and C have Cartesian coordinates (3, - 10, 7) and (2, - 7, 9) and (2, - 8, 7) respectively, relative to an origin O. find

(a) $AB \times AC$

(b) the area of triangle OAB,

(c) a vector and Cartesian equation of the plane ABC

(d) the volume of the tetrahedron OABC

(e) the distance from O to the plane ABC

8. (i) Define the concept of homomorphism between two groups G and G/. The function f defined by $f : G \rightarrow G', x \mapsto e'$ where e' is the identity element of G/. Show that f is a homomorphism from G to G/.

(ii) Sketch, on the same diagram, the curves C_1 and C_2, defined by the polar equations

$$C_1 : r\cos\theta = 2, \frac{-\pi}{2} < \theta \le \frac{\pi}{2} \qquad C_2 : r = \frac{6}{4\cos\theta}, -\pi < \theta < \pi$$

(a) State the polar coordinates of the point of intersection of C_1 with the initial line.

(b) The curve C_2 intersects the initial line at the point A and the half lines $\theta = \frac{\pi}{2}$ and $\theta = -\frac{\pi}{2}$ at the points B and C respectively. Find the polar coordinates of A, B and C.

9. (i) A curve C_1 has equation $2y^2 = x$. Another curve C_2 is given parametrically as $x = 4t, y = \frac{4}{t}$. Sketch C_1 and C_2 on the same diagram and calculate the coordinates of the point of intersection.

(ii) Sketch the curve $y = f(x)$, where $f(x) = \frac{2x^2}{9-4x}$, showing clearly the asymptotes, turning points and intercepts with the coordinate axes.