A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2004 MEETLEARN.COM

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A Level Further Pure Maths

1. Points A, B and C have coordinates (6, 3, 9), (2, 8, 10) and (4, 12, 12), respectively, referred to Cartesian axis $OXYZ$. Find
(a) $AB \times AC$
(b) the vector and Cartesian equations for the plane ABC.
(c) The volume of the tetrahedron OABC
Another plane contains the points A, B and D (7, 4, 10).
Find the cosine of the acute angle between the planes ABC and ABD.

2. (i) Solve the differential equation $\left(1-x^2\right)\frac{dy}{dx}+xy=\left(1-x^2\right)^{\frac{1}{2}}e^{\cos x}\sin x$.
(ii) Given that $A\cos 4x + B\sin 4x$ is a particular integral of the second order differential equation $y''+9y=14\sin 4x$, find the values of the real constants A and B and find the general solution of the differential equation. Find, also, the solution for which $y=2$ and $\frac{dy}{dx}=7$ when x = 0.

3. Given the ellipse E with equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that the equation of the normal to E at the point $P(a\cos\varphi, b\sin\varphi)$ is $\frac{ax}{\cos\varphi}-\frac{by}{\sin\varphi}=a^2-b^2$.
The normal at P to E meets the x – axis at M and the y – axis at N. show that the locus of Q, the mid-point of MN, is another ellipse E_1. Show, further, that the eccentricity of E_1 is the same as that of E.

4. (i) Find the fifth roots of unity and show that they form an Abelian group under multiplication.
(ii) Verify that the set C of complex numbers under usual addition and multiplication forms a field.

5. A linear transformation $T:\Re^3 \to \Re^3$ maps x to $T(x)$. The vectors a, b, c are mapped respectively to p, q, r by this transformation, where

$$a=\begin{pmatrix}1\\1\\0\end{pmatrix};\ b=\begin{pmatrix}2\\1\\0\end{pmatrix};\ c=\begin{pmatrix}1\\2\\2\end{pmatrix};\ p=\begin{pmatrix}3\\4\\4\end{pmatrix};\ q=\begin{pmatrix}-5\\7\\6\end{pmatrix};\ r=\begin{pmatrix}8\\9\\-8\end{pmatrix}.$$

Find the matrix T which represents this transformation.
Find T^{-1}, and hence, find the image of the point which maps to (2, 3, -5)

6. (i) Show that

$$\frac{1+\tanh^2 3x}{1-\tanh^2 3x} \equiv \cosh 6x.$$

By using the substitution $t = \tanh 3x$, or, otherwise, find $\int \sec h 6x dx$.

(ii) Solve for real x, the equation $e^{\sinh^{-1}x} - 1 = e^{\cosh^{-1}x}$.

(iii) Show that

$$\int_0^{\ln 2} \frac{dx}{5\cosh x - 3\sinh x} = \frac{\pi}{8} - \frac{1}{2}\tan^{-1}\frac{1}{2}$$

7. (i) Use De Moivre's theorem to show that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$.

Deduce that $\cos^2\frac{\pi}{8} = \frac{2+\sqrt{2}}{4}$

(ii) Show that, if $f(z) = z^6 - 2z^3\cos 3\alpha + 1$, then $z^3 = 3\cos\alpha \pm i\sin 3\alpha$ if $f(z) = 0$. By using De Moivre's theorem, express $f(z)$ as a product of quadratic factors.

8. (i) Find the length of the arc of the curve $x = \ln t, 2y = \left(1 + \frac{1}{t}\right)$, between the points (0, 1) and $\left(\ln 2, \frac{5}{4}\right)$

(ii) Given that $n \in Z^+$ and $I_n = \int_0^{\pi} e^{-x}\sin^n x dx$.

Show that for $n > 1$, $I_n = \frac{n}{1-n} I_{n-2}$

9. (i) sketch, using different axes, the curves with polar equations;

(a) $r = 2 + \sin\theta$ (b) $r = 2\cos 2\theta$

In (b), find the equations of the tangents at the pole.

(ii) Show that, $\frac{1}{2} \le \frac{x^2+x+7}{x^2-4x+5} \le 13\frac{1}{2}$, and sketch the graph of the function

$f(x) = \frac{x^2+x+7}{x^2-4x+5}$, indicating how the curve approaches its asymptotes.

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