A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2012 MEETLEARN.COM

A Level Further Pure Maths

1. (a) Solve the differential equation $x\frac{dy}{dx} - y = x^2 \cos x$, given that y = 0 when x = π.

(b) Find the constants A and B such that $A\cos 2x + B\sin 2x$ is a particular integral of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 2\sin 2x$.

Hence, obtain the general solution of the differential equation.

Find, also, the solution for which $y = 3$ and $\frac{dy}{dx} = 2$ when x = 0.

2. (a) Express $f(x) = \frac{x^2}{(x^2+1)^2}$ in partial fractions.

Hence, using the substitution $x = \tan\theta$ or otherwise, prove that $\int_0^1 f(x)dx = \frac{1}{8}(\pi - 2)$.

(b) Given that $I_n = \int_1^{e^2} (\ln x)^n dx$, show that $I_n = 2^n e^2 - nI_{n-1}$. Hence, evaluate I_3

3. (a) When x^5 and higher powers are neglected,

$\ln\left(\frac{1-\sinh x}{1+x}\right) \approx ax + bx^3 + cx^4$. Find the values of the real constants a, b and c.

(b) Find the Maclaurin series expansion of $\cos x^2$ as far as the term in x^8.

Show that the general term, U_n, of this expansion can be written as $U_n = \frac{(-1)^n x^{4n}}{(2n)!}$.

Hence, show that the series is convergent for all real values of x.

4. (a) The point P in the Argand diagram represents the complex number z, and Q represents the complex number ω, where $\omega = \frac{i}{z-i}$.

Given that P lies on the circle with centre at the origin and radius 1 unit,

(i) Prove that Q lies on the curve $|\omega| = |\omega + i|$

(ii) Sketch the locus represented by $|\omega| = |\omega + i|$.

(b) Find the roots of the equation $(z-4)^3 = 8i$, giving your answer in the form a + bi, where a and b are real numbers. Indicate, on an Argand diagram, the points A, B, C representing these roots. Find the area of triangle ABC.

5. Show that the length of the curve $8(y+\ln x)=x^2$ between x = 1 and x = e is $\frac{1}{8}\left(7+e^2\right)$.

Find the area of the surface of revolution obtained by rotating this curve through 2π radians about x – axis. Using a theorem of Pappus, find the y coordinate of the centroid of this curve.

6. (a) Find the real values of x for which $8\cosh x + 4\sinh x = 7$, giving your answer in terms of natural logarithms.

(b) Use the definition of $\coth x$ in terms of exponential functions to prove that

$$\coth^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right), x^2 > 1$$

A Function f is defined by $f(x) = \coth^{-1}\left(\frac{x}{2}\right), x^2 > 4$.

(i) Show that $f'(x) = -\frac{2}{x^2-4}$

(ii) Expand $f(x)$ as a series in ascending powers of $\frac{1}{x}$ as far as the term in $\frac{1}{x^5}$.

7. (a) Given the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$, find the value(s) of λ for which $|A - \lambda I| = 0$, where I is a unit matrix.

(b) Show that the transformation T, represented by the matrix

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & -2 \\ 3 & -2 & -7 \end{pmatrix},$$ maps the whole space onto the plane $x - 2y + z = 0$.

Find the image under this transformation of

(i.) the line $x = -y = \frac{z-1}{2}$,

(ii.) the plane $x - y - z = 0$

8. (a) It is given that x is an element of a group G with identity element e and that $x^3 = x$. Show that $x^2 = e$.

(b) Consider the groups G_1, G_2, G_3, where

$G_1 = (\{1,3,7,9\}, \times_{10})$,

$G_2 = (\{1,5,7,11\}, \times_{12})$,

$G_3 = (\{1,3,5,7\}, \times_8)$,

where $\times_n$ means multiplication modulo n.

(iii) Draw up group tables for G_1, G_2, G_3

(iv) Find which of the two groups are isomorphic and write down an isomorphism between them.

(v) Solve the equation $x^3 = x$ in each of the three groups.

9. The lines L_1 and L_2 are given by

$L_1 : r = 3i + j + 2k + \lambda(i + 2j - k)$

$L_2 : r = 6i - j + k + \mu(-2i + k)$

The plane Π contains the lines L_1 and L_2. Find

(a) the position vector of the point of intersection of L_1 and L_2;

(b) a vector normal to the plane Π;

(c) a Cartesian equation of the plane Π;

(d) the distance of the point (3, -1, 4) from the plane Π;

(e) the position vector of the point of intersection of the line $\frac{x-1}{1} = \frac{y+2}{-3} = \frac{z-2}{2}$ and the plane Π