A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2013 MEETLEARN.COM

A Level Further Pure Maths

1. Given the curve C such that

$y=\sqrt{(1-x^{\frac{2}{3}})^3}$, for $0 \le x \le 1$.

Show that

(i) the length of the curve C is $\frac{3}{2}$

(ii) the point $(\sqrt{t^3}, \sqrt{(1-t)^3})$ lies on the curve C.

The curve C is rotated completely about the x-axis.

Find

(iii) The area of the surface of revolution obtained.

(iv) Using a theorem of Pappus, the y-coordinate of the centroid of the arc C.

2. (a) using the substitution y=vx, where v is a function of x, show that the differential equation

$x^2\frac{dy}{dx}=xy-y^2$ can be transformed into the form $x\frac{dv}{dx}+v^2=0$.

Hence, find the general solution of the differential equation

$x^2\frac{dy}{dx}=xy-y^2$ in the form $y=f(x)$

(b) Given that $y=Ae^x\cos 5x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2}+25y=e^x(\cos 5x-10\sin 5x)$$

Find the value of the constant A.

Hence solve completely the differential equation given that $y=3, \frac{dy}{dx}=-4$ when x=0.

3. The points A,B and C have Cartesian coordinates (2,-1,4), (10,7,2) and (0,0,6) relative to the origin O.

Find,

(i) ABxAC

(ii) the area of triangle ABC

(iii) the length of the perpendicular line from the point b to the line ac.

(iv) the Cartesian equation of the plane ABC

(v) the volume of the tetrahedron ABCD, given that the plane ABC cuts the x-axis at D.

4. (a) the polar curve C, has equation given by $C: r=\sqrt{3}+2\cos\theta$

Find the tangents to the curve at the pole.

Sketch the curve C.

(b) Given that $f(x)=\frac{2x^2}{3-2x}, x\neq\frac{3}{2}$

(i) Show that $f(x)$ cannot take values between 0 and-6, for real values of x.

(ii) Sketch the curve $y=f(x)$, showing clearly the intercepts, turning points and the behavior of the curve near its asymptotes.

5. (a) Test whether or not the following series are convergent.

(i) $\sum_{n=1}^{\infty} \frac{4^n}{3^{2n-1}}$

(ii) $\sum_{n=1}^{\infty} \frac{3n}{\sqrt{2n^3+1}}$

(b) Express $17\cosh x + 15\sinh x$ in the form $R\cosh(x+\ln a)$, where R and a are positive integers.
Hence, otherwise

(i) Solve the equation $17\cosh x + 15\sinh x - 40 = 0$.

(ii) Find the maximum value of $\frac{56}{34\cosh x + 30\sinh x + 12}$

6. (I) given that $x^2 - y^2 = 0$, find the inverse of the matrix A, where $A = \begin{pmatrix} x & 0 & y \\ 0 & 1 & 0 \\ y & 0 & x \end{pmatrix}$

(ii). Show that the set S, of all matrices of the form $\begin{pmatrix} a & 0 & b \\ 0 & 1 & 0 \\ b & 0 & a \end{pmatrix}$,

a, b $\in$R, $a^2 \neq b^2$ forms a group under matrix multiplication.

(iii) Given the sets M and N of matrices of the forms

$\begin{pmatrix} 0 & 0 & x \\ 0 & 1 & 0 \\ x & 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} y & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & y \end{pmatrix}$, respectively, $x, y \in \mathbb{R}, x \neq 0, y \neq 0$

Determine with reasons, whether or not M and N are subgroups of S.
(Assume associativity of matrix multiplication)

7. (a) Express $f(x)$, where $f(x) = \frac{2x+3}{(x+1)^2(x+2)^2}$, in partial fractions.

Hence, or otherwise show that $\int_1^3 f(x)dx = \frac{7}{60}$

(b) Given that $I_n = \int_{-1}^{1} (1+x)^n \sqrt{1-x}\,dx, n \geq 1$

Show that $(2n+3)I_n = 4nI_{n-1}, n \geq 1$ Hence, find I_1

8. (a) Express $\sin 2\theta$ *and* $\cos 2\theta$ in the form $e^{2i\theta}$ *and* $e^{-2i\theta}$

Hence, or, otherwise, show that $16\sin^2 2\theta\cos^3 2\theta = 2\cos 2\theta - \cos 6\theta - \cos 10\theta$

Evaluate $\int_0^{\frac{\pi}{3}} 16\sin^2 2\theta \cos^3 2\theta\, d\theta$

(b) A complex transformation T from the z- plane to the w-plane is given by

$T: \omega = \frac{z-2}{\ldots}$

Hence, or otherwise show that $\int_1^3 f(x)dx = \frac{7}{60}$

(b) Given that $I_n = \int_{-1}^{1} (1+x)^n \sqrt{1-x}\,dx, n \geq 1$

Show that $(2n+3)I_n = 4nI_{n-1}, n \geq 1$ Hence, find I_3

8. (a) Express $\sin 2\theta$ *and* $\cos 2\theta$ in the form $e^{2i\theta}$ *and* $e^{-2i\theta}$

Hence, or, otherwise, show that $16\sin^2 2\theta \cos^3 2\theta = 2\cos 2\theta - \cos 6\theta - \cos 10\theta$

Evaluate $\int_0^{\frac{\pi}{3}} 16\sin^2 2\theta \cos^3 2\theta\, d\theta$

(b) A complex transformation T from the z- plane to the w-plane is given by

$$T; \omega = \frac{z-2}{z-i}$$

Show that the image of the circle $|z| = 2$ is also a circle in the $\omega -$ *plane* and sketch it.

9. (i) prove that the equations of the tangent and normal to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $P(a\cos\theta, b\sin\theta)$ are respectively

$bx\cos\theta + ay\sin\theta = ab$ and $ax\sin\theta - by\cos\theta = (a^2 - b^2)\sin\theta\cos\theta$.

(*ii*) Given that the tangent cuts the $x -$ *axis* at A and the $y -$ *axis* at B and that the normal cuts the $x -$ *axis* at C and the $y -$ *axis* at D. Show that as θ varies the locus of the midpoint of CD is

$$4a^2x^2 + 4b^2y^2 = (a^2 - b^2)^2.$$

Given that $a = 5$ *and* $b = 4$, sketch this locus