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Mathematics Paper 2, 2016 Past GCE

JUNE 2016

1. (i) Given that the roots of the equation $x^2 - x + 2 = 0$ are α and β, find the quadratic equation whose roots are $\frac{1}{1+\alpha^2}$ and $\frac{1}{1+\beta^2}$.

(ii) Given that the polynomial
$$P(x) = (2x - 1)(x - 3)Q(x) + 12x - 8,$$
of degree 3, is exactly divisible by $x - 1$ and that $P(0) = 10$, find $Q(x)$.

(10 marks)

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2. Given the matrices **M** and **N**, where
$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & 1 \\ 5 & 1 & 0 \end{pmatrix} \text{ and } \mathbf{N} = \begin{pmatrix} -1 & 0 & 1 \\ 5 & 0 & -2 \\ 6 & 3 & -3 \end{pmatrix},$$
find matrix product **MN** and **NM**.
Hence find $\mathbf{M}^{-1}$, inverse of **M**.
The transformation represented by the matrix **M** maps the points A, B and C to the points $(3, 0, 6)$, $(0, 5, 3)$ and $(1, 0, 1)$ respectively.
Find the coordinates of A, B and C.

(8 marks)

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3 (i) Given that the function $f(x) = x^3$ is differentiable in the interval $(-2, 2)$, use the mean value theorem to find the value of x for which the tangent to the curve is parallel to the chord through the points $(-2, 8)$ and $(2, 8)$.

(ii) Express in the form $y = f(x)$, the general solution of the differential equation $y\frac{dy}{dx} = x(1 + y^2)$.

(12 marks)

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4. (i) Use De moivre's theorem to express $\cos 4\theta$ in terms of $\cos\theta$.
(ii) Given that $z_1 = 2 + i$, $z_2 = -2 + 4i$ and $\frac{1}{z_3} = \frac{1}{z_1} + \frac{1}{z_2}$, find z_3.

(12 marks)

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5. The position vectors of the points A, B and C are **a**, **b** and **c** respectively, where
$$\mathbf{a} = 3\mathbf{i} + 6\mathbf{k}, \mathbf{b} = 5\mathbf{j} + 3\mathbf{k} \text{ and } \mathbf{c} = \mathbf{i} + \mathbf{k}.$$
Find
(i) the vector product $\overrightarrow{AC} \times \overrightarrow{AB}$,
(ii) the vector equation of the plane ABC.

(8 marks)

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6. Express $f(\theta) = 8\cos 4\theta - 15\sin\theta$ in the form $r\cos(\theta + \alpha)$, where r is positive and α an acute angle.
Hence, find
(i) the general solution of the equation $80\cos\theta - 150\sin\theta = 13$,
(ii) the maximum and minimum value of $\frac{5}{f(\theta)+3}$.

(9 marks)

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7. (i) Express $f(x) = \frac{5x-3}{(x+1)(x+3)}$, in partial fractions.
Hence, evaluate $\int_3^5 f(x)dx$.

(ii) Given that $f(x) = 5x^2 - 4\sqrt{x} - 6$, $x > 0$ and taking 1.5 as a first approximation to the root of $f(x) = 0$, use the Newton-Raphson procedure to obtain, to three decimal places, a second approximation to the root of the equation.

(10 marks)

8. (i) Find the set of values of x for which $\frac{x+2}{x-1} < 3$.

(ii) Given the function f, where $f(x) = \frac{3x-4}{x+2}$, $x \neq -2$,
(a) find the range of f.
(b) sketch the graph of f.

(10 marks)

9. (In this question, you are required to work throughout with two decimal places.)

The table below shows the values of x and y obtained in a certain laboratory work.

x	2.659	4.801	6.248	9.708	17.595	20.96
y	10.317	6.569	5.63	4.512	3585	3.38

The variables x and y are connected by the equation $y - 2 = b(x-1)^a$.
By drawing a suitable graph of $\log_{10}(y-2)$ against $\log_{10}(x-1)$, estimate the values of the constants a and b.

(10 marks)

10. (i) Find the term independent of x in the expansion of $(x^2 + \frac{1}{2x})^6$.

(ii) A geometric progression with positive terms has the sum of its first two terms as $\frac{20}{3}$ and its sum to infinity is 12.
Find
(a) the first term and the common ratio of the progression,
(b) the sum of the first three terms.

(12 marks)