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Mathematics Paper 3, 2016 Past GCE

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1. (i) The position vector of a particle P of mass 6 kg at any time t seconds is given by

$$\mathbf{r} = \left[\cos\left(\frac{t}{2}\right)\mathbf{i} + \sin\left(\frac{t}{2}\right)\mathbf{j}\right] \text{m}.$$

(a) Show that the position and acceleration vectors of the particle are in opposite direction.
(b) Show that the velocity and the acceleration are always at right angles to each other.

Given that $t = \pi$, find
(c) the magnitude of the momentum of the particle.

(ii) At the time $t = 0$ the position vectors of two paticles R and Q are respectively $(5\mathbf{i} + 20\mathbf{j})$ m and $(-10\mathbf{i} - 5\mathbf{j})$ m. The particles have constant velocities $(-2\mathbf{i} - 5\mathbf{j})$ m s^{-1} and $(3\mathbf{i} + 5\mathbf{j})$ m s^{-1} respectively.
(d) Find the position vector of Q relative to R after 3 seconds.

2.

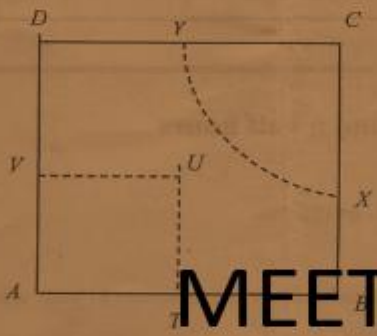

Fig. 1

Fig. 1 shows a uniform square lamina $ABCD$ of sides $6a$ metres . A portion of the lamina in the form of a square $ATUV$, where T and V are respectively the midpoints of AB and AD, is removed. Another portion of the lamina in the form of a sector XCY of center C, where X and Y are the midpoints of BC and CD, is also removed.
(a) Show, by integration, that the centre of gravity of the uniform lamina in the form of a sector XCY is a distance $\frac{4a}{\pi}$ metres from the bounding radii.
(b) Find the distance of the centre of gravity of the remaining lamina from AD.

3. (i) A uniform ladder AB of length 6 m and mass 2 kg rest with its upper end B against a smooth vertical wall and the lower end A on rough horizontal ground. The ladder makes an angle of 30^0 with the horizontal ground. Given that the ladder is about to slip, find the angle of friction.

(ii)

Fig. 2

In Fig. 2, the particles A and B, of mass 8 kg and 6 kg respectively, are connected by a light inextensible string which passes over a smooth pulley fixed at C. A lies on a rough horizontal plane and B lies on a smooth plane inclined at 45^0 to the horizontal. The coefficient of friction between A and the plane is μ. The system is in limiting equilibrium.
Find the value of μ.
Given that B is replaced with another particle of mass 8 kg, find
(a) the acceleration of the system,
(b) the tension in the string.

(Take g as10 ms^{-2})

4. A smooth sphere P, of mass m kg, moves on a horizontal smooth table with speed u ms^{-1} and collides w an identical sphere Q, lying at a distance 4 m from a vertical wall. Sphere Q subsequently collides normally with the wall and rebounces. If the coefficient of restitution between any impact is $\frac{1}{2}$, find

(a) the velocities of P and Q after the first impact,

(b) the velocity of Q after collision with the wall.

Prove that the next collision between P and Q will take place at a distance 1.6 m from the wall.

5. (i) A car starts from rest at a point A and accelerates uniformly at $3x$ ms^{-2} until it reaches a speed of 30 ms^{-1}. It maintains this speed for the next t seconds and retards uniformly at x ms^{-2} to rest at the B. The distance between A and B is 6 km and the time taken from A to B is 5 minutes.

(a) Show that $\frac{40}{x} + 2t = 400$.

(b) Find the values of t and x.

(ii) A particle moves along a straight line so that at time t s its speed v ms^{-1} and its distance x m from a point O on the line are given by

$$v = \frac{1}{2}(5x + 3) \text{ ms}^{-1}.$$

Given that initially the particle was at O, show that the time taken by the particle to move a distance from O is $\frac{4}{5}\ln 4$.

6. Two forces $\mathbf{F_1}$ and $\mathbf{F_2}$, where

$$\mathbf{F_1} = (-4\mathbf{i} + \mathbf{j}) \text{ N and } \mathbf{F_2} = (\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) \text{ N},$$

act at the points with position vectors $(a\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ m and $(-2\mathbf{i} + a\mathbf{j} - \mathbf{k})$ m respectively.

Find

(a) the equations of the line of action of $\mathbf{F_1}$ and $\mathbf{F_2}$ in terms of a.

(b) the value of the constant a given that the lines of action of $\mathbf{F_1}$ and $\mathbf{F_2}$ intersect.

7. (i) The engine of a car of mass 500 kg works at a constant rate of 15 kW. A non-gravitational resistanc the motion of the car is constant. The maximum speed of the car on level ground is 5 ms^{-1}.

(a) Find the acceleration of the car when it is travelling at a speed of $\frac{5}{2}$ ms^{-1} directly up a road of inclination α, where $\sin\alpha = \frac{1}{10}$.

Given that the car covers a distance of 20 m up the road,

(b) Calculate the work done by the engine

(ii) A particle of mass 5 kg moves on the inside surface of a smooth spherical bowl of radius 4 m, descri a horizontal circle at a distance 2 m below the centre of the bowl with uniform speed.

Find

(c) the speed of the particle,

(d) the force acting towards the centre.

(Take g as 10 ms^{-2}.)

8. (i) Two boxes are labelled A and B. Box A contains 4 red marbles and 2 blue marbles. Box B contains and 4 blue marbles. An experiment consists of throwing a fair die and selecting a marble. If a prime number shows on the die, a marble is selected from box A, otherwise a marble is selected from box Draw a probability tree diagram showing the possible outcomes from the experiment.

Using the tree diagram, or otherwise, find the probability that

(a) a red ball is selected,

(b) a blue ball is chosen from box B.

(ii) The events P and Q are such that $\text{P}(P) = \frac{2}{5}$, $\text{P}(P \cap Q) = \frac{2}{7}$ and $\text{P}(P|Q) = \frac{4}{7}$.

Find

(c) $\text{P}(Q)$,

(d) $\text{P}(Q|P)$,

(e) determine whether the events P and Q are independent.